Lenth's method

Assume $Z \sim N(0, \tau^2)$. Then the median of $|Z|$ $\approx 0.675\,\tau$. Thereby $1.5 \cdot$ Median of $|Z| \approx \tau$.

Let $\hat{\theta}_1, \ldots, \hat{\theta}_m$ be the estimators of the effects.

An estimator for their standard deviation if all were 0 is $\hat{\tau} = 1.5 \cdot \text{Median}(|\hat{\theta}_i|, i = 1, 2, \ldots, m)$

If some are different from zero

$$\hat{\tau}^{\circ} = 1.5 \ \text{Median}(|\hat{\theta}_i|;\ |\hat{\theta}_i| < 2.5\,\hat{\tau})$$

For testing $H_0: \theta_i = 0 \quad H_1: \theta_i \neq 0$

we use that $T = \dfrac{\hat{\theta}_i}{\hat{\tau}^{\circ}}$ is approximately t-distributed with $m/3$ degrees of freedom.

Use of higher order interactions to test for significance of effects

Often we assume that three-factor and higher order interactions are zero.

In a 2^4 experiment there are 4 three-factor interactions: ABC, ABD, ACD and BCD and 1 four-factor interaction ABCD.

If these interactions are zero, they represents 5 observations with expected value = 0 and equal variances. Hence $\hat{\sigma}^2_{effect} = \dfrac{\hat{ABC}^2 + \hat{ABD}^2 + \hat{ACD}^2 + \hat{BCD}^2 + \hat{ABCD}^2}{5}$

The T statistic then becomes, $T = \dfrac{\hat{Effect}}{\hat{\sigma}_{effect}}$ and an effect is declared significant if $|T_{obs}| \geq t_{\frac{\alpha}{2}, 5}$

Estimation of variance by replication

Example

A	B	C	y_{i1}	y_{i2}	$y_{i1} - y_{i2}$	$\frac{(y_{i1} - y_{i2})^2}{2}$
−	−	−	59	61	−2	2
+	−	−	74	70	4	8
−	+	−	50	58	−8	32
+	+	−	69	67	2	2
−	−	+	50	54	−4	8
+	−	+	81	85	−4	8
−	+	+	46	44	2	2
+	+	+	79	81	−2	2
Total						64

For each i; $\sum_{j=1}^{2} (y_{ij} - \bar{y}_i)^2 = \left(y_{i1} - \frac{y_{i1} + y_{i2}}{2}\right)^2 + \left(y_{i2} - \frac{y_{i1} + y_{i2}}{2}\right)^2$

$$= \left(\frac{y_{i1} - y_{i2}}{2}\right)^2 \cdot 2 = \frac{(y_{i1} - y_{i2})^2}{2}$$

An estimate for σ^2, $s^2 = \frac{64}{8} = 8$

An estimate for σ^2_{effect}; $s^2_{effect} = \frac{4 \cdot 8}{16} = 2$ and has 8 degrees of freedom.

Interpretation of effects

Factors not involved in interactions with others.

Estimated main effect represent the estimated change in expected response when we go from low to high level of the corresponding factor

Factors involved with interactions:

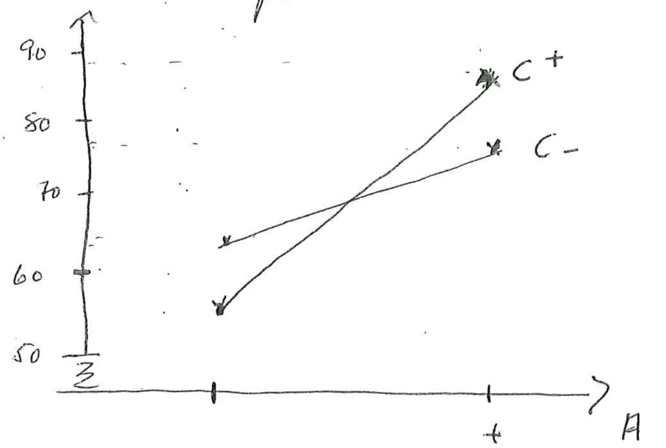
Then the impact of a factor on the response will depend on the level of the other factors.

2^3 experiment. Cracks in springs.

$\hat{A} = 23$, $\hat{AC} = 10$

		C	
A	−	$\frac{67+61}{2} = 64$	$\frac{59+52}{2} = 55.5$
	+	$\frac{79+75}{2} = 77$	$\frac{90+87}{2} = 88,5$

A graphical visualization of this table is called an interaction plot



The effect of A is much larger when C is on high level compared to C is on low level, The largest responsevalue is obtained when both A and C are on high levels.